

Spin noise in polariton lasers

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We develop a theory of spin fluctuations of exciton-polaritons in a quantum microcavity under the non-resonant unpolarized pumping. It is shown that the spin noise is sensitive to the scattering rates in the system, occupation of the ground state, statistics of polaritons and interactions. The spin noise spectrum drastically narrows in the polariton lasing regime due to formation of a polariton condensate, while its shape becomes non-Lorentzian owing to interaction-induced spin decoherence.

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Introduction. Quantum microcavity is a system where a semiconductor quantum well is placed between the Bragg mirrors making it possible to achieve the strong coupling between the light and matter. In this system the energy is coherently transferred back and forth between the photon trapped in the microcavity and the exciton, the elementary excitation of the semiconductor. First observed by Weisbuch *et al.* [1], the strong coupling results in the formation of mixed light-matter quasiparticles, exciton-polaritons, extensively studied since then [2, 3]. The exciton-polaritons combine an extremely small effective mass, inherited from the photon, and strong interactions between themselves and with the environment due to the excitonic fraction in this quasiparticle, allowing one to study plenty of fascinating phenomena including polariton lasing [4], which consists in accumulation of a huge number of polaritons in a single quantum state (non-equilibrium condensate) and spontaneous emission of coherent light by this state.

The exciton-polaritons are characterized by the spin projections $s_z = \pm 1$ onto the structure growth axis z corresponding to the right or left circular polarization of the photon and to the same spin component of the exciton. Superpositions of $s_z = \pm 1$ polariton states give rise to the linear or elliptical polarization of exciton-polaritons. Polariton lasers represent a model bosonic system where the polarization of light emitted by a microcavity directly corresponds to the spin state of exciton-polaritons, allowing one to study the polariton spin dynamics by optical methods [5]. A wealth of prominent spin-related effects were proposed and realized in microcavities, including self-induced Larmor precession [6, 7], linear polarization inversion [8], optical spin Hall effect [9–11], spin Meissner effect [12–14], spin multistability [15, 16], etc., see Refs. [2, 17] for reviews.

In polariton lasers, the spontaneous symmetry breaking results in the appearance of a stochastic vector polarization including circular polarization directly proportional to the spin of a polariton condensate [18, 19]. The Stokes vector (pseudospin) of a condensate may be pinned to one of the crystal axes [20], due to the structure anisotropy. In this case the time-averaged polarization of emission of

polariton lasers is defined by pinning, while its momentary value may fluctuate. The spin fluctuations give rise to the “spin noise” which may be studied experimentally by optical means. The spin fluctuations, being inherent to any system at equilibrium or not, were first observed in 1981 in Na vapor [21]. Nowadays they became a research field in semiconductor spintronics, since measurements of their properties can provide an important information on spin dynamics, hardly accessible otherwise [22–27].

Here we study theoretically the spin noise spectra of polariton lasers both below and beyond the laser threshold and demonstrate that it is extremely sensitive to the mean occupation number of the condensate and to the statistics of polaritons. Our approach can be easily extended to a rich variety of bosonic systems where spin polarization may be formed including cold atoms, indirect excitons, triplet superconductors and ^3He superfluids.

Model. The dynamics of the exciton-polariton spin doublet can be described with the pseudospin approach where the density matrix of polariton state with the wavevector \mathbf{k} can be written as $\hat{\rho}_{\mathbf{k}} = N_{\mathbf{k}}\hat{I} + \mathbf{S}_{\mathbf{k}} \cdot \hat{\boldsymbol{\sigma}}$. Here \hat{I} is the unit 2×2 matrix, $\hat{\boldsymbol{\sigma}}$ is a pseudovector composed of Pauli matrices, $N_{\mathbf{k}}$ is the spin-average occupancy of the state \mathbf{k} , and $\mathbf{S}_{\mathbf{k}}$ is the pseudospin of polariton in this state. In what follows we focus on the dynamics of the ground state, corresponding to $\mathbf{k} = 0$ and treat all other states in the system as a reservoir. Correspondingly, we omit \mathbf{k} subscript in the notations. The pseudospin components S_{α} , where $\alpha = \{x, y, z\}$ is the Cartesian index, correspond to the polarization of emission: S_z/N gives its circular polarization degree, while S_x/N and S_y/N give the linear polarization degree in the axis frames (xy) and $(x'y')$ rotated by 45° with respect to each other. [2] The dynamics of the occupation number and the polariton pseudospin is governed by the set of kinetic equations [28]

$$\frac{dN}{dt} + \frac{N}{\tau_0} + Q_n\{N, \mathbf{S}\} = 0, \quad (1)$$

$$\frac{d\mathbf{S}}{dt} + \mathbf{S} \times \boldsymbol{\Omega} + \frac{\mathbf{S}}{\tau_0} + \mathbf{Q}_s\{\mathbf{S}, N\} = 0, \quad (2)$$

where τ_0 is the lifetime of polaritons in the ground state, $\boldsymbol{\Omega}$ is the effective magnetic field arising from (i) anisotropy

of the system and (ii) from polariton-polariton interactions, its explicit form will be given below. The scalar, $Q_n\{N, \mathbf{S}\}$, and pseudovector, $\mathbf{Q}_s\{\mathbf{S}, N\}$, collision integrals describe arrival and departure of the particles into the ground state. The general form of the collision integrals is presented in Ref. [28] for the polariton-phonon interaction and in Ref. [6, 29] for the polariton-polariton interaction. Here we adopt their simplest form

$$\begin{aligned} Q_n\{N, \mathbf{S}\} &= W^{\text{out}}N - W^{\text{in}}(1 + N), \\ \mathbf{Q}_s\{\mathbf{S}, N\} &= (W^{\text{out}} - W^{\text{in}} + \tau_s^{-1})[\mathbf{S} - \mathbf{S}_0(\mathbf{\Omega})]. \end{aligned} \quad (3)$$

Here W^{out} and W^{in} are out- and in-scattering rates, related with the presence of the reservoir, as schematically illustrated in Fig. 1. In particular, W^{in} is proportional to the occupation of the reservoir and it is determined by the pumping rate. The term proportional to $1 + N$ reflects the bosonic nature of exciton-polaritons. It describes the stimulated scattering processes. In the collision integral for the polariton spin, $\mathbf{Q}_s\{\mathbf{S}, N\}$, τ_s is the spin relaxation time and $\mathbf{S}_0(\mathbf{\Omega})$ is the steady-state spin induced by the effective field $\mathbf{\Omega}$. If polariton exchange with reservoir is efficient, one can introduce the effective temperature T of the polariton system. The steady-state spin is $-\langle N \rangle \mathbf{\Omega} / |\mathbf{\Omega}|$ for $T \ll \Omega$ and 0 for $T \gg \Omega$. In what follows (unless otherwise is specified), we consider the case of high temperatures, where $\langle \mathbf{S} \rangle \equiv 0$, and the occupancy of the polariton ground state given by the balance of in- and out- scattering processes:

$$\langle N \rangle = \frac{W^{\text{in}}}{\tau_0^{-1} + W^{\text{out}} - W^{\text{in}}}, \quad (5)$$

where the condition $W^{\text{in}} < \tau_0^{-1} + W^{\text{out}}$ should hold [30].

The fluctuations of the condensate occupation number $\delta N(t) \equiv N(t) - \langle N \rangle$, and pseudospin $\delta \mathbf{S}(t) \equiv \mathbf{S}(t) - \langle \mathbf{S} \rangle = \mathbf{S}(t)$ are described by the correlation functions, namely: $\mathcal{K}(t) \equiv \langle \delta N(t') \delta N(t' + t) \rangle$ and $\mathcal{C}_{\alpha\beta}(t) \equiv \langle S_\alpha(t') S_\beta(t' + t) \rangle$, where the angular brackets denote the averaging over the time t' for a given t . According to the general theory of fluctuations [31–33] the correlation functions of the fluctuations as functions of t or t' obey the same set of kinetic equations as fluctuating quantities [34]. The solution of Eqs. (1) and (2) in the absence of effective magnetic fields and interactions ($\mathbf{\Omega} \equiv 0$), results in the exponential time-decay of correlations and isotropic spin fluctuations:

$$\mathcal{K}(t) = \mathcal{K}(0)e^{-|t|/\tau_c}, \quad \mathcal{C}_{\alpha\beta}(t) = \delta_{\alpha\beta} \mathcal{C}_{\alpha\alpha}(0)e^{-|t|/\tau_{c,s}}, \quad (6)$$

where $\delta_{\alpha\beta}$ is the Kronecker δ -symbol, single time correlators (mean square fluctuations) $\mathcal{K}(0) = \langle (\delta N)^2 \rangle$, $\mathcal{C}_{\alpha\alpha}(0) = \langle S_\alpha^2 \rangle_0$ will be found below, while the particle-spin correlations $\langle \delta N(t') S_\alpha(t' + t) \rangle$ vanish and will be disregarded. We introduced the correlation times τ_c , $\tau_{c,s}$ according to

$$\frac{1}{\tau_c} = \frac{1}{\tau_0} + W^{\text{out}} - W^{\text{in}} = \frac{\tau_0^{-1} + W^{\text{out}}}{1 + \langle N \rangle}, \quad \frac{1}{\tau_{c,s}} = \frac{1}{\tau_c} + \frac{1}{\tau_s}. \quad (7)$$

Equations (6) and (7) clearly show that the particle number and spin fluctuations of exciton-polaritons decay exponentially as a function of time difference $|t|$ and the

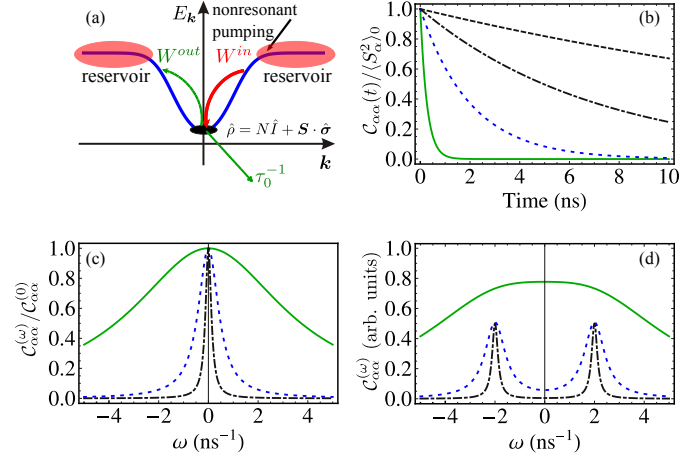


FIG. 1: (a) Illustrative scheme of the pumping. The reservoir and the ground state are shown as well as in- and out- scattering processes. (b) Temporal dependence of spin fluctuations calculated for the $\langle N \rangle = 10$ (green/solid), 100 (blue/dotted) 1000 (black/dash-dotted). Other parameters are: $\tau_0 = 25$ ps, $W^{\text{out}} = 0$, $\tau_s = 10$ ns. Dashed curve is calculated for $\tau_s \rightarrow \infty$ and $\langle N \rangle = 1000$. (c) Spin noise power spectra calculated for the same parameters. (d) Spin noise power spectra calculated with allowance for the anisotropic splitting $\Omega_a = 2$ ns⁻¹. Other parameters are the same as in panels (b), (c).

correlation time of the particle number fluctuations is τ_c , while the spin fluctuations are correlated on a shorter time scale $\tau_{c,s}$. The spin fluctuation spectra defined as $\mathcal{C}_{\alpha\beta}^\omega \equiv \int_{-\infty}^{\infty} \mathcal{C}_{\alpha\beta}(t) e^{i\omega t} dt$ have a Lorentzian form:

$$\mathcal{C}_{\alpha\alpha}^\omega = \mathcal{C}_{\alpha\alpha}(0) \frac{2\tau_{c,s}}{1 + \omega^2 \tau_{c,s}^2}, \quad (8)$$

with the half-width half maximum $\tau_{c,s}^{-1}$ determined by the inverse spin correlation time. It follows from Eq. (7) that the correlation time of fluctuations is strongly enhanced and the spin noise spectrum is strongly narrowed, if $\langle N \rangle \gg 1$, i.e. where the ground state is macroscopically occupied. In particular, if $\tau_s \rightarrow \infty$ and $W^{\text{out}} \rightarrow 0$ which corresponds to the negligible spin-flip in the ground state and negligible depletion of the condensate due to the scattering of polaritons back to the reservoir, we have for the correlation time of particle and spin fluctuations $\tau_c = \tau_0(1 + \langle N \rangle)$. The life-time of exciton-polaritons in the state-of-the-art structures varies from ~ 1 ps to ~ 100 ps with the corresponding maximum $\langle N \rangle$ being between 10^3 and 10^5 , yielding τ_c in the range of 1 ns ... 10 μ s. Correspondingly, the spin noise frequencies range from MHz to GHz for the macroscopically occupied ground state. The typical spin noise spectra and temporal dependence of the spin fluctuations are presented in Fig. 1(b),(c) where the drastic effect of the ground state occupation is clearly visible. The several orders of magnitude enhancement of the spin correlation time and corresponding narrowing of the noise spectrum is a purely bosonic effect, inherent to exciton-polaritons. Indeed, any fluctuation (in the particle number or in the spin) is supported by the bosonic stimulation due to $1 + N$ factor in the in-scattering term of the collision integral in Eq. (3).

The mean square of the particle and pseudospin fluctuations can be found using the approach of Ref. [35] (see also Ref. [3]) where the statistics of the exciton-polariton ground state was found by means of the master equation approach. Assuming that interactions and effective magnetic field are absent, the system becomes spin-isotropic and can be described by the independent occupations of two orthogonal spin states, N_\uparrow and N_\downarrow , described by the same distribution functions $P(N_{\uparrow,\downarrow})$. Using these distribution functions one can express the mean square fluctuations, that is $t = 0$ correlators $\langle(\delta N)^2\rangle = \sum_{N_\uparrow, N_\downarrow} P(N_\uparrow)P(N_\downarrow)[(N_\uparrow + N_\downarrow)/2 - \langle N \rangle]^2$ and $\langle S_\alpha^2 \rangle_0 = \sum_{N_\uparrow, N_\downarrow} P(N_\uparrow)P(N_\downarrow)[(N_\uparrow - N_\downarrow)/2]^2$, as

$$\langle S_\alpha^2 \rangle_0 = \langle(\delta N)^2\rangle = \frac{1}{2}\langle N \rangle[1 + (g^{(2)} - 1)\langle N \rangle], \quad (9)$$

where $\langle S_\alpha^2 \rangle_0$ corresponds to isotropic fluctuations, and $g^{(2)}$ is the second order coherence of a single state. In particular, $g^{(2)} = 2$ corresponds to the thermal statistics [36], where particle and spin square fluctuations are $\propto \langle N \rangle(1 + \langle N \rangle)$ and grow quadratically with the ground state occupation. This situation is realized for equilibrium Bose gas [37] or if the ground state feedback on reservoir is negligible. By contrast, $g^{(2)} = 1$ corresponds to the coherent statistics, in which case the fluctuations are suppressed, being $\propto \langle N \rangle$. In the limit of low occupancy of the ground state, $\langle(\delta N)^2\rangle^{1/2} = \sqrt{\langle N \rangle}$ in agreement with the theory of classical gas. Moreover, the full statistics of spin fluctuations can be determined, as a convolution of distribution functions $P(N - S_\alpha)$ and $P(N + S_\alpha)$. We present here the result in the $\langle N \rangle \gg 1$ limit:

$$p_{\text{coh}}(S_\alpha) = (\pi\langle N \rangle)^{-1/2} \exp(-S_\alpha^2/\langle N \rangle), \quad g^{(2)} = 1, \quad (10)$$

$$p_{\text{th}}(S_\alpha) = \langle N \rangle^{-1} \exp(-2|S_\alpha|/\langle N \rangle), \quad g^{(2)} = 2. \quad (11)$$

For pulsed excitation, the stochastic polarization of polariton condensates was studied in Ref. [19] by numerical integration of Langevin equations for the condensate spinor wavefunctions. The approach of Ref. [19] can be extended to a continuous wave pumping making it possible, in principle, to determine the full statistics of the system. Here we resort to a simpler analytical approach based on kinetic equations treating $g^{(2)}$ as an external parameter.

Role of effective magnetic fields. We begin the discussion of the effective magnetic fields with the case of an anisotropic system where the polariton doublet is split into the pair of states, linearly polarized along x and y axes. In such a case the vector $\mathbf{\Omega} = (\Omega_a, 0, 0)$ determines the anisotropic splitting [6, 38–40], and we assume that the effective temperature T of the system exceeds the anisotropic splitting to neglect the steady spin polarization. The S_z and S_y show precession, while the dynamics of S_x remains purely dissipational. As a result, the fluctuations become anisotropic with nonzero spectrum components (c.f. Ref. [41])

$$\frac{\mathcal{C}_{xx}^{(\omega)}}{\langle S_\alpha^2 \rangle_0} = \frac{2\tau_{c,s}}{1 + \omega^2\tau_{c,s}^2}, \quad (12)$$

$$\frac{\mathcal{C}_{yy}^{(\omega)}}{\langle S_\alpha^2 \rangle_0} = \frac{\tau_{c,s}}{1 + (\omega - \Omega_a)^2\tau_{c,s}^2} + \frac{\tau_{c,s}}{1 + (\omega + \Omega_a)^2\tau_{c,s}^2}, \quad (13)$$

$$\mathcal{C}_{yz}^{(\omega)} = \frac{2i\omega\Omega_a\tau_{c,s}^2}{1 + \tau_{c,s}^2(\omega^2 + \Omega_a^2)}\mathcal{C}_{yy}^{(\omega)}, \quad (14)$$

where $\mathcal{C}_{zz}^{(\omega)} = \mathcal{C}_{yy}^{(\omega)}$, $\mathcal{C}_{yz}^{(\omega)} = [\mathcal{C}_{zy}^{(\omega)}]^*$, $\langle S_\alpha^2 \rangle_0$ is given by Eq. (9), and $\tau_{c,s}$ by Eq. (7). Figure 1(d) shows the spin noise spectra calculated with the allowance for the anisotropic splitting for different occupations of the ground state. It is seen that the single peak is transformed to the two peak structure. This is because the spin correlation time $\tau_{c,s}$ depends strongly on the ground state occupation: for small pumping rates and small $\langle N \rangle$ the product $\Omega_a\tau_{c,s} \ll 1$ and the splitting is not visible, however, for larger pumping $\Omega_a\tau_{c,s}$ becomes comparable or larger than 1, making the anisotropic splitting $\hbar\Omega_a$ resolvable in the spin noise, as demonstrated in Fig. 1(d). If the temperature is so low that $T \ll \hbar\Omega_a$, the polaritons are predominantly polarized along the effective field direction x . In this case, $\langle S_x \rangle = -\langle N \rangle$, $\langle S_y \rangle = \langle S_z \rangle = 0$, while the mean square fluctuations take the form $\langle(S_x - \langle S_x \rangle)^2\rangle = \langle(\delta N)^2\rangle$, $\langle S_y^2 \rangle = \langle S_z^2 \rangle = \langle N \rangle/2$. Moreover, the non-trivial single time correlation appears: $\langle S_y S_z \rangle = \langle S_z S_y \rangle^* = -i\langle N \rangle/2$. Spin noise spectrum in this case can be obtained in a similar way. Specifically, Eqs. (12), (13) still hold, but with different numerators corresponding to mean square fluctuations found above.

Now let us consider the effect of polariton-polariton interactions on the spin noise. As follows from multiple experimental and theoretical works [8, 12–14, 16, 42] these interactions are strongly spin-anisotropic: the exciton-polaritons with the same z pseudospin components, i.e. with the same circular polarizations, repel efficiently due to the exchange interaction of electrons/holes with the same spin, while the polaritons with opposite circular polarizations can weakly attract each other [8, 42], the latter effect is neglected here. Hence, the interactions create an effective fluctuating field $\mathbf{\Omega} = (0, 0, \Omega_i)$, directed along the z axis. Here the magnitude $\hbar\Omega_i \equiv \alpha_1 S_z$ ($\alpha_1 > 0$) is related to the fluctuations of polaritons pseudospin z component. This effective field has two important consequences on polariton spin dynamics and spin noise: (i) it induces precession of the pseudospin around the z axis, known as self-induced Larmor precession [6, 7], and (ii) it suppresses fluctuations of the z pseudospin component, favoring linear polarization of the macrooccupied state [12, 43].

It is instructive to start the analysis with the spin precession effect assuming that the effective temperature is high enough, $T \gg \alpha_1\sqrt{\langle S_z^2 \rangle_0}$. We also ignore in what follows the anisotropic splitting, taking $\Omega_a = 0$. Clearly, the effective field Ω_i results in the dephasing of the in-plane pseudospin components. To describe the effect quantitatively we note, that according to Eq. (2) the correlation functions of the in-plane pseudospin components are governed by the following equation:

$$\frac{d\mathcal{C}_{\alpha\beta}(t)}{dt} + \frac{\mathcal{C}_{\alpha\beta}(t)}{\tau_{c,s}} \pm \frac{\alpha_1}{\hbar}\mathcal{C}_{\alpha\beta}(t)S_z(t) = 0. \quad (15)$$

Here the upper (lower) sign corresponds to $\alpha = x$ ($\alpha = y$),

correlations between z - and in-plane components are disregarded because the effective field Ω does not couple them, and, under the above assumptions, S_z can be considered as an independent parameter whose fluctuations are given by Eqs. (6), (8). The set of linear equations (15) can be readily solved as:

$$\frac{C_{\alpha\alpha}(t)}{\langle S_\alpha^2 \rangle_0} = e^{-|t|/\tau_{c,s}} \left\langle \exp \left[i \frac{\alpha_1}{\hbar} \int_0^{|t|} S_z(t_1) dt_1 \right] \right\rangle_z, \quad (16)$$

with $\langle \dots \rangle_z$ meaning the averaging over the fluctuations of S_z . The treatment of the general case is beyond the scope of the present work, here we consider two limiting cases: the regimes of (a) fast and (b) slow fluctuations.

If $\alpha_1 \langle S_z^2 \rangle_0^{1/2} \tau_{c,s} / \hbar \ll 1$, the change of the effective field takes place on the time scale, which is much smaller than the spin precession period in the interaction induced field Ω_i . This case corresponds to the motional narrowing and

$$\frac{C_{\alpha\alpha}(t)}{\langle S_\alpha^2 \rangle_0} = \exp \left(-\frac{|t|}{\tau_{c,s}} - \frac{\alpha_1^2}{\hbar^2} \langle S_\alpha^2 \rangle_0 \tau_{c,s} |t| \right). \quad (17)$$

In this limit the spin fluctuations decay exponentially, resulting in the Lorentzian shape of the spin noise power spectrum. This spectrum is, however, anisotropic: the half-width half maximum of $C_{xx}^{(\omega)}$ and $C_{yy}^{(\omega)}$ is larger than that of $C_{zz}^{(\omega)}$ because the interaction-induced field does not affect S_z owing to the spin-anisotropy of polariton-polariton interactions.

In the opposite limit, where the fluctuations of spin z component are slow enough, so that the in-plane pseudospin components make several turns during the correlation time of S_z , i.e. if $\alpha_1 \langle S_z^2 \rangle_0^{1/2} \tau_{c,s} / \hbar \gg 1$, the fluctuations of S_z and, hence, the effective field Ω_i can be considered frozen. In this case, the spin dephasing takes place on the timescale of Ω_i^{-1} . The particular t -dependence of the correlator $C_{\alpha\alpha}(t)$ is determined by statistics of the condensate [44]. For $\langle N \rangle \gg 1$ we obtain for two important limiting cases of coherent and thermal statistics ($t \ll \tau_{c,s}$):

$$\frac{C_{\alpha\alpha}(t)}{\langle S_\alpha^2 \rangle_0} = \exp(-\Gamma_{\text{coh}}^2 t^2), \quad g^{(2)} = 1, \quad (18)$$

$$\frac{C_{\alpha\alpha}(t)}{\langle S_\alpha^2 \rangle_0} = \frac{1}{1 + \Gamma_{\text{th}}^2 t^2}, \quad g^{(2)} = 2, \quad (19)$$

where the dephasing rates are: $\Gamma_{\text{coh}}^2 = \alpha_1^2 \langle N \rangle / 4\hbar^2$ and $\Gamma_{\text{th}}^2 = \alpha_1^2 \langle N \rangle^2 / 4\hbar^2 = \langle N \rangle \Gamma_{\text{coh}}^2$. In this limit, the temporal dependence of the spin fluctuations is directly related to the ground state statistics: Gaussian fluctuations of S_z described by $p_{\text{coh}}(S_z)$ in Eq. (10) result in the Gaussian decay of the in-plane spin components, while the sharply-peaked exponential distribution $p_{\text{th}}(S_z)$ in Eq. (11) for thermal statistics, results in the slow power-law decay of the fluctuations due to high probability of small values of S_z , corresponding to low precession rates. As a result, the noise spectrum of the in-plane pseudospin components deviates strongly from Lorentzian. Making Fourier trans-

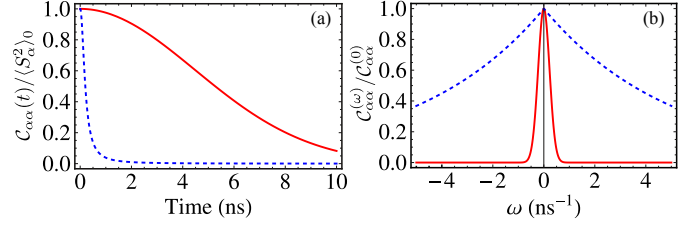


FIG. 2: (a) Temporal dependence of the in-plane pseudospin fluctuations calculated with allowance for interactions according to Eqs. (18) and (19) for $\langle N \rangle = 1000$, $\alpha_1 = 10^{-2} \text{ ns}^{-1}$ [42] for coherent statistics (red/solid) and thermal statistics (blue/dotted). (b) Spin noise power spectra calculated for the same parameters as in (a).

form of Eqs. (18), (19) we obtain:

$$\frac{C_{\alpha\alpha}^{(\omega)}}{\langle S_\alpha^2 \rangle_0} = \frac{\sqrt{\pi}}{\Gamma_{\text{coh}}} \exp(-\omega^2 / 4\Gamma_{\text{coh}}^2), \quad g^{(2)} = 1, \quad (20)$$

$$\frac{C_{\alpha\alpha}^{(\omega)}}{\langle S_\alpha^2 \rangle_0} = \frac{\pi}{\Gamma_{\text{th}}} \exp(-|\omega| / \Gamma_{\text{th}}), \quad g^{(2)} = 2. \quad (21)$$

It is noteworthy, that the non-analytical dependence of the spin fluctuations for the thermal statistics at $\omega \rightarrow 0$ is related with the power-law temporal decay of the fluctuations. The typical temporal dependence of the in-plane pseudospin fluctuations and noise power spectra are plotted in Fig. 2(a) and (b), respectively. Figure clearly shows different qualitative behavior of the noise of the in-plane pseudospin components for different statistics of the polaritons. We emphasize that the drastic difference of the decoherence times and noise spectral widths for coherent and thermal statistics is related to different dependences of $\langle S_z^2 \rangle$ on the ground state occupancy, $\langle N \rangle$, see Eq. (9).

Note, that if $\alpha_1 \langle S_z^2 \rangle$ is comparable with or larger than the effective temperature T of the system, the fluctuations of the pseudospin z component would be suppressed. This effect can be modeled by multiplying the distribution function of S_z in Eqs. (10) and (11) by the Boltzmann factor $\exp(-\hbar\alpha_1 S_z^2 / T)$ for the probability of thermal fluctuations [12]. As a result, in the limit of $T \rightarrow 0$ we obtain for $\langle S_z^2 \rangle = T / 2\hbar\alpha_1$. In this case, the dephasing rate of the in-plane pseudospin components, which determines the spin noise spectral width, can be estimated as $\sim \sqrt{\alpha_1 T / \hbar}$.

Conclusions. A theory of spin fluctuations in polariton lasers had been developed. We have demonstrated that the spin noise of exciton-polaritons strongly depends on the occupation numbers, statistics and interactions in the polariton condensate, thus providing a tool for the studies of a large variety of bosonic spin effects. Various regimes of spin noise have been identified. Experimental verification of these predictions can be done by Fourier spectroscopy of Kerr or Faraday rotation spectra. Extension of this model to other spin-polarised bosonic systems is straightforward.

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